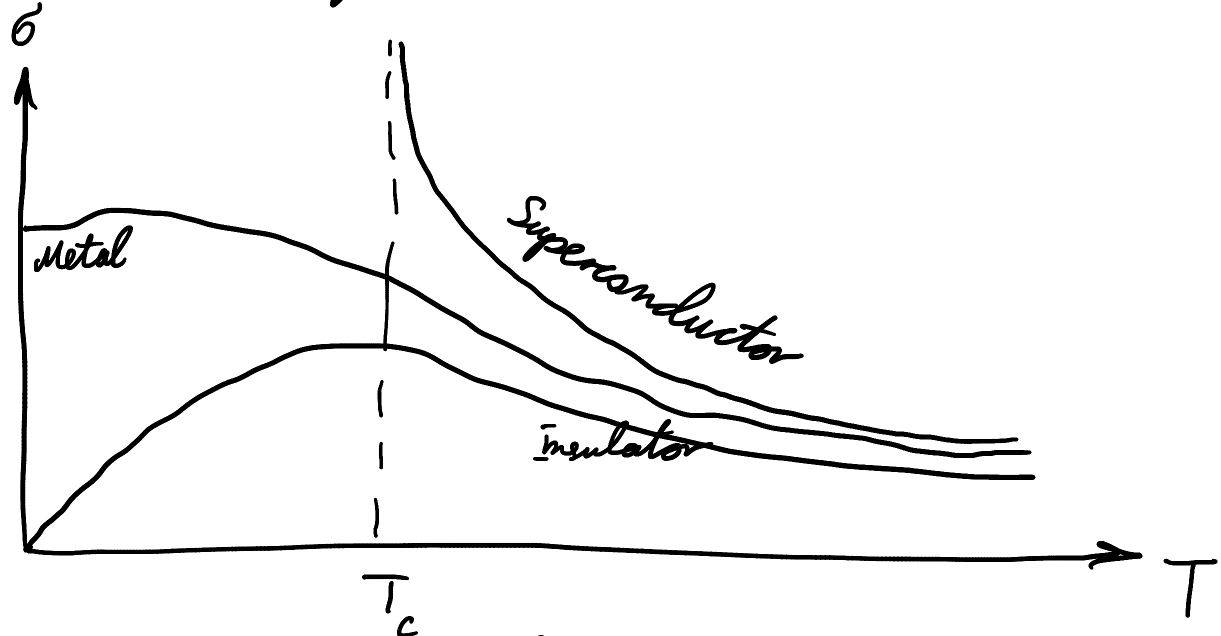
Anderson localisation

All systems may be divided into three classes depending on the behaviour of conductivity at $T=0$



Previously we focussed on metals, showing by direct calculations that $\sigma(T=0)>0$

Often it's possible to induce a transition between a metal and an insulator by changing the amount of disorder or the interaction strength

Anderson transition

Mott transition

In general, Mott-Anderson transition

Let us focus on the case of a non-interacting disordered system.

$$\left(-\frac{\hbar^2 \Delta}{2m} + U(\vec{r})\right)\Psi(\vec{r}) = E\,\Psi(\vec{r})$$

Random potential

<u>Infinite system</u>

There may be two types of states

1) Extended, $\int |\Psi(\vec{r})|^2 d\vec{r} = \infty$

2) Localised, $\int |\Psi(\vec{r})|^2 d\vec{r} = 1$

(normaliseable states)

Examples:

*) In a clean system $(U=0)$, $\Psi = e^{i\vec{k}\vec{r}}$

In a smooth potential $U(x)$

$$\Psi \sim \frac{1}{\sqrt{v(x)}}\, e^{\frac{i}{\hbar}\int p(x)\,dx}$$

$p(x) = \sqrt{2m(E-U(x))}$

$v(x) = \frac{p(x)}{m}$

for quadratic spectrum

*) Particle in a potential well

1D: $$-\frac{\hbar^2}{2m}\partial_x^2\Psi - \alpha\,\delta(x)\,\Psi = E\Psi$$

$$\Psi = A\,e^{-\varkappa|x|}, \text{ with } \varkappa = \sqrt{-\frac{2mE}{\hbar^2}}$$

$$-\frac{\hbar^2}{2m}\Psi'(+0) + \frac{\hbar^2}{2m}\Psi'(0-) - \alpha\Psi(0) = 0$$

$$\frac{\hbar^2}{m}\sqrt{-\frac{2mE}{\hbar^2}} = \alpha \longrightarrow E = -\frac{m\alpha^2}{2\hbar^2}$$

Bound states exist in arbitrarily shallow potential wells in 1D and 2D, but in 3D the well has to be sufficiently deep

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Localised states do not contribute to conduction

$$\langle v\rangle = \int \Psi^*(\vec r)\,\hat v_{\vec r}\,\Psi(\vec r)\,d\vec r = -\frac{i}{m}\int \Psi^*(\vec r)\,\vec\nabla\,\Psi(\vec r)\,d\vec r$$

In the case of a localised state, the wavefunction $\Psi(\vec r)$ may be chosen real

$$\langle v\rangle = -\frac{i}{2m}\int \vec\nabla\left[\Psi(\vec r)\right]^2 d\vec r = 0$$

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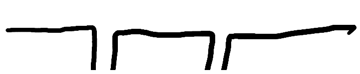
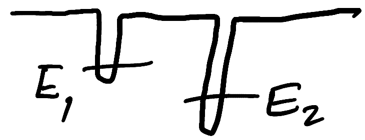
* *) Two potential wells



when isolated, energies E_1 and E_2



E_1 and E_2 when close to each other, some tunnelling from one well to the other may occur

$$\hat{H} = \begin{pmatrix} E_1 & I \\ I & E_2 \end{pmatrix} \qquad E = \frac{E_1+E_2}{2} \pm \left(I^2 + \left(\frac{E_1-E_2}{2}\right)^2\right)^{\frac{1}{2}}$$

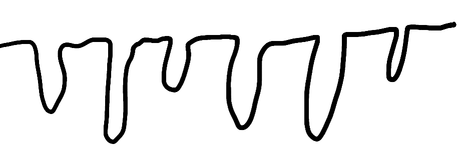
Wavefunctions $\psi = c_1\varphi_1 \pm c_2\varphi_2$

$$\frac{c_1}{c_2} = \frac{I}{\frac{E_1-E_2}{2} \pm \left[\left(\frac{E_1-E_2}{2}\right)^2 + I^2\right]^{\frac{1}{2}}}$$

When $|E_1 - E_2| \gg I$, $\frac{c_1}{c_2} \to \infty, 0$, i.e. the particle is sitting in one of the two wells

When $|E_1 - E_2| \ll I$, the particle spreads equally between the 2 wells, $\psi \approx \frac{\varphi_1 \pm \varphi_2}{\sqrt{2}}$

If there are many wells

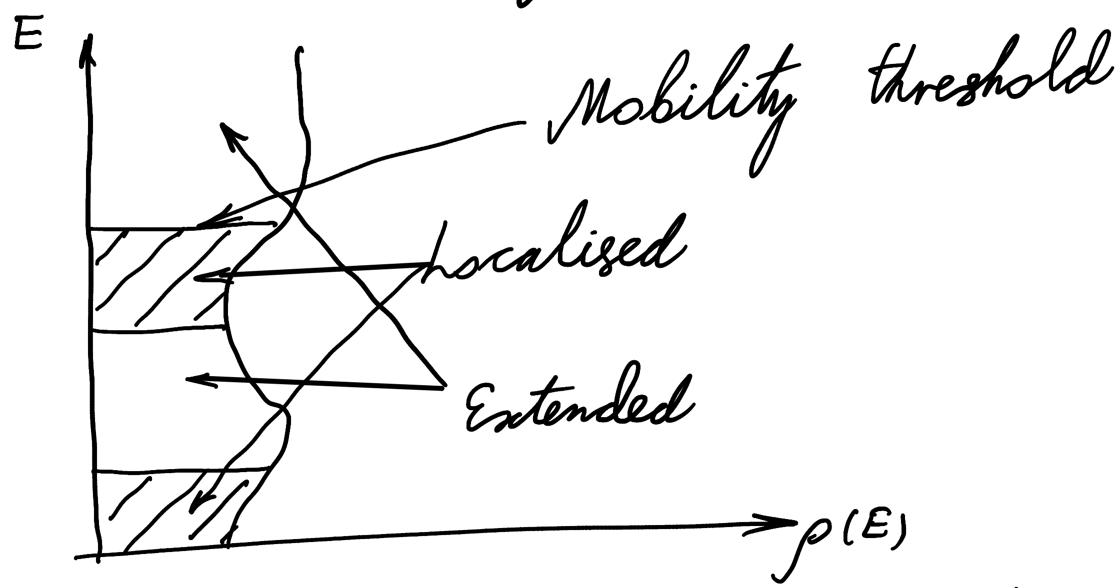


with $|E_i - E_j| \lesssim I$, one may expect that that they will hybridise and form a extended state.

that they ...
state.

In fact, for a system of potential wells extended states appear only in 3D
In 1D and 2D everything is localised

In general, ∃ both localised and delocalised states — they do not coexist at the same energy — Mott's argument



When changing disorder strength, mobility thresholds move.

Localised states do not contribute to conductivity.

Mott's hypothesis about minimal metallic

Mott's hypothesis about minimal metallic conductivity:

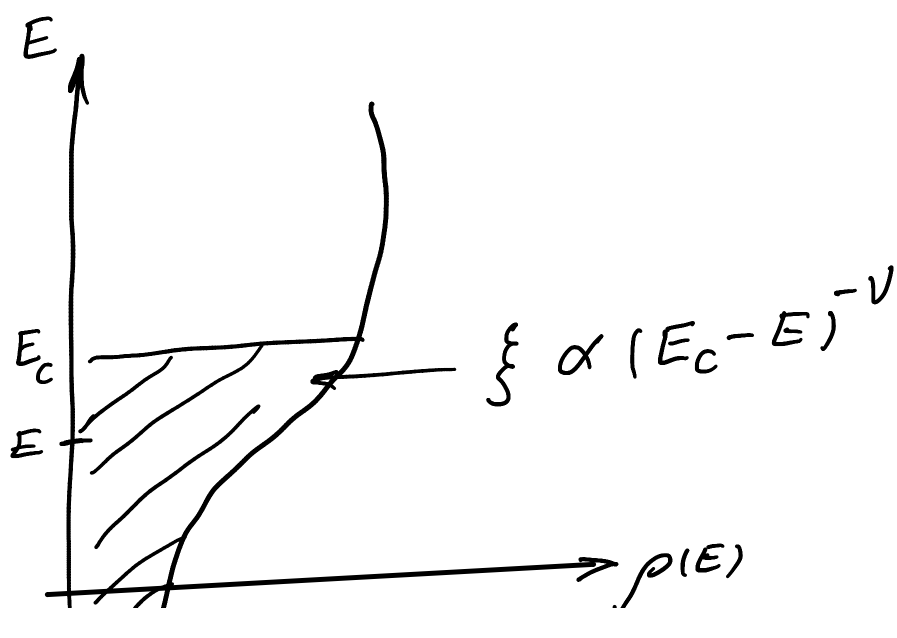
$$\sigma \sim \frac{e^2}{\hbar}(k_F l)\, k_F$$

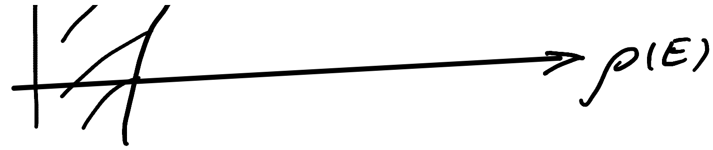
The minimal conductivity would correspond to $k_F l \sim 1$. $\sigma_{min} \sim \frac{e^2}{\hbar} k_F$. Then he conjectured it to drop to zero.

However, Anderson localisation transition is <u>continuous</u>

When approaching the mobility threshold,

$$\xi \sim |E - E_c|^{-\nu}$$ ← universal exponent





$$E_c = E_c(x)$$

Disorder strength in a given material

Now let us fix E and change disorder strength.

$$E \approx E_c + \frac{dE}{dx}(x - x_c)$$

$$\boxed{\xi \propto (x_c - x)^{-\nu}}$$

How the localisation length changes when changing disorder strength. In principle, there are similar dependencies for all parameters.